

Internet-Based Teaching and Learning in a Mid-Size Honors Multivariable Calculus Course

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Abstract

At the 2005 international symposium, Enhancing University Mathematics at KAIST in Daejeon, Korea, we reported on our ten-year experience of teaching geometry and calculus courses at all levels using the Internet both for communication and for demonstrations [B1] At the 2007 ATCM conference in Taipei, we presented an example of an interactive article on Critical Points and Curvature based on these courses and this student-developed software [B2]. The purpose of this article is to update those two reports and to present evidence that interactive Internet-based courses can enhance teaching and learning at different scales. The primary examples come from a course at Brown University in the fall semester of 2008 on honors multivariable calculus for a class of 66 students, twice the number as in previous classes. An extensive questionnaire at the conclusion of that class provides comparative data that validate the effectiveness of some earlier modifications and suggest new directions for pedagogical research.

This report contains four sections:

- 1—Summary of Features of the Tensor for Internet-Based Teaching and Learning
- 2—The Two-Dimensional Table of Contents and Improvements in Laboratory Software
- 3—Examples of Student Work Using the Tensor and Demonstration Software
- 4—Assessment and Future Directions Based on Comparison of Questionnaire Results

1—Summary of Features of the Tensor for Internet-Based Teaching and Learning

Several key features of our approach have been developed primarily in relatively small classes of approximately 30 students. This past semester, 66 students signed up for that same course so the scale was different. This necessitated rethinking several aspects of our courseware, some of which were readily adaptable to a larger group and several of which open new research questions.

Math 35 has two 80-minute lectures per week, a format more suited to an honors course where students generally have more background and a longer attention span than to a standard course where three 50-minute classes per week spreads out the new ideas in a way that is more easily

digestible. For a class of 30 students, it is possible to assign and respond to two assignments per week, with responses posted online in the Tensor. The first assignment is given out on the afternoon of the Tuesday class and is to be handed in online by Wednesday night at 10 p.m. This short assignment gives the students a chance to see if they have understood the lecture material well enough to solve problems, as well as to think about the new ideas that are going to be presented on Thursday. The benefit for the teacher is intermediate feedback that indicates whether or not it is safe to continue with an idea that most students have followed up to that point. It also flags certain points of confusion that can be cleared up at the beginning of the Thursday lecture. After that lecture, a more substantial problem set is posted, due on Monday morning by 3 a.m. so that the online responses are ready to be read and commented on by the instructor (and assistants if available) by Monday afternoon in preparation for the upcoming week. Up to the time the assignment is due, only the student can view his or her work and the instructor comments, and after the due time, all students in the class can read the responses of others, together with comments, unless some submissions have been designated as “private” (a feature instituted as a result of requests on questionnaires of previous courses).

The Tensor is the piece of communication software where all the responses are collected. By default only the last five assignments are presented in a matrix, and selecting "Show All Assignments" will present all the entries of all students over the course of the semester. Selecting a week or other heading in the leftmost column will open a matrix for that assignment, with problems listed on the leftmost column and student user id initials across the top. Selecting any entry shows the student's response and the comment of the instructor. A square in the matrix is red if the most recent response came from the student and green if the most recent activity is a comment from the instructor. Mathematical autobiographies are automatically private, i.e. so they are only available to be read by the instructors of that class.

One (unintended) consequence of the ability to respond to comments is that some students engage in a dialogue until they get all the aspects of a problem correct. While this is certainly beneficial to the student, it involves extra time on the part of the instructor. Even without such back-and-forth online discussion, the practice of providing responses before the beginning of the next class involves a serious time commitment on the part of the instructor.

Before class on Tuesday, students are expected to read the comments on their work as well as the Solution Key, usually a compilation of exemplary (or at least interesting) responses transferred from student responses by the instructor, often together with additional commentary. It is possible for students to enter comments about the work of others in the class, although this aspect is not often used unless the instructor specifically requests it on certain occasions.

This commitment on the part of the students and the instructor is substantial and labor intensive. For 30 students, it is possible to keep up the effort; for 66, it is far more difficult. There are two major differences that emerged in the course in the fall of 2008: First of all, students still were faithful and appreciative about reading comments on their own work, but fewer took the opportunity to look at the work of other students in the Tensor, relying instead on the selected responses collected in the Solution Key. Secondly, to a much greater extent than previously, the course employed group work. Each of these aspects was treated in the end-of-course

questionnaire, and the assessment based on these data appears in the final section of this paper, where we also refer to suggestions and comments about the privacy option and the Solution Key. One aspect that did not change was student use of the demonstration software in assignments and in the interactive laboratories. Under the Resources Menu in the course webpage, there is a link to the interactive laboratories developed using the senior thesis of David Eigen, a math-CS concentrator who devised the program for generating Java applets for use in a variety of mathematics courses at Brown. The Math 35 page includes a basic tutorial for using the Java demos. Each section of the labs starts with a labeled picture and a plus sign that can be selected to see more information about the concept illustrated in the demo as well as a button that opens a Java applet. The student can explore a particular phenomenon, entering in new data and manipulating the images. In order to share what he or she has discovered or created, the student can save the applet tag and enter it into a homework answer or a discussion. Anyone who opens the button that appears in the document will enter the program exactly where the student left off. The instructor or another student can then make comments, sometimes illustrated by a modification of the given applet.

A difference from earlier instances this course is the variety of techniques used by students to enter their responses in the Tensor. As a result of student suggestions and requests from earlier courses, we included “shortcuts” to make it easier for students to type mathematical expression in html. A number of students continued to use this feature throughout the semester, while others chose to use more sophisticated software, primarily LaTeX. We also made it more straightforward for students to upload their written work and many chose to submit their work this way, both in assignments and in examinations. It is probably worthwhile repeating the comment made in previous papers that students can immediately type their work into the Tensor without any significant preparation or instruction, once they know how to enter exponents and subscripts and a few mathematical symbols for inequalities and the square root. For example, in the first assignment after the first lecture, Thunwa Theerakarn typed in his answer to the following problem:

Problem: What is the range of the function $f(x,y) = x^2 + 2bxy + y^2$, defined for all (x,y) ? (The answer will depend on b . Try to cover all possible cases.)

Thunwa Theerakarn at 2008-09-07 17:07:43.0

To find the range of the function $f(x,y) = x^2 + 2bxy + y^2$, for any y , we can choose $t = x + by$. Then $f(x,y) = t^2 + (1 - b^2) y^2$

Case $|b| \leq 1$:

Since $t^2 \geq 0$ and $(1 - b^2) y^2 \geq 0$, we have $f(x,y) \geq 0$.

Range is all positive real numbers and zero.

Case $|b| < 1$:

Since t^2 can be any positive real number and $(1 - b^2) y^2$ can be any negative real number, range of $f(x,y)$ is all real numbers.

Another feature of the online course that worked well but required extra effort was giving take-home examinations. There were two mid-term exams, each scheduled for five days, and a one-week final examination. Such exams routinely include a number of standard problems, as well as others that are less specifically defined and more open to interpretation and exploration. It takes a long time to grade such examinations, since the instructor is responsible for all such grading. Some examples of more complex problems that can be assigned in such examinations are given in the next section. In the author's viewpoint, these more challenging and open-ended problems can be the most rewarding aspect of the class, for the students as well as the instructor. They give students a chance to display a range of thoughtful responses, often leading to multiple approaches to problems that can be collected in the Solution Key.

Examples of such open problems and student responses appear in the third section of this report.

2—The Two-Dimensional Table of Contents and Improvements in Laboratory Software

A new feature of our laboratory software is a more explicit two-dimensional table of contents. Calculus topics appear in the rows of a matrix, with three columns, for functions of one, two, or three variables. At any point, a student can navigate from a topic in two-variable calculus “to the left” to recall the corresponding one-variable topic in a way suitable for generalization, or “to the right” to see further generalizations for functions of three and more variables. We continue to look for ways to make this access tool even more powerful and engaging. Each topic listed in the two-dimensional table of contents is in a column according to its dimension. At any time, a student can move “leftward” from a laboratory demonstration on two-variable calculus to the corresponding demonstration for one-variable calculus, or “rightward” to see how the notion will be extended to functions of three or more variables.

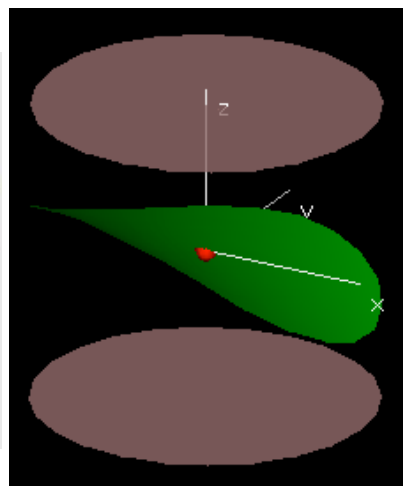
1D Tutorial 2D Tutorial Course List	1D	2D	3D
Functions	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Polar Coordinates <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Spherical Coordinates <input type="checkbox"/> Cylindrical Coordinates <input type="checkbox"/>
Derivatives	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Polar Coordinates <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Spherical Coordinates <input type="checkbox"/> Cylindrical Coordinates <input type="checkbox"/>
More Derivatives	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Polar Coordinates <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/>
Integration	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Polar Coordinates <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Spherical Coordinates <input type="checkbox"/> Cylindrical Coordinates <input type="checkbox"/>
Vector Analysis	Rectangular Coordinates <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/>	Rectangular Coordinates <input type="checkbox"/>

1D Tutorial 2D Tutorial Course List	1D	2D	3D
Functions	Rectangular Coordinates Linear Functions Domain, Range & Graphs Slice Curves Level Sets & Contours Color Domain Continuity <input type="checkbox"/> Parametric Equations <input type="checkbox"/>	Rectangular Coordinates Linear Functions Domain, Range & Graphs Slice Curves Arbitrary Slice Curves Level Sets & Contours Color Domain Continuity <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Polar Coordinates <input type="checkbox"/>	Rectangular Coordinates Linear Functions Domain, Range & Graphs Slice Curves Arbitrary Slice Curves Slice Surfaces Level Sets & Contours Color Domain Continuity <input type="checkbox"/> Parametric Equations <input type="checkbox"/> Spherical Coordinates <input type="checkbox"/> Cylindrical Coordinates <input type="checkbox"/>

We illustrate this setup with two examples. The first involves continuity, a notion that many students understand only imperfectly after a first course in calculus, especially from a geometric point of view that can be particularly helpful in calculus of two variables. We interpret continuity as a challenge-response situation, and in any number of variables, the process is the same. Given a function $f(x)$ of one variable and a point x_0 of the domain, the challenger chooses a pair of horizontal lines at distance ε from $y = f(x_0)$ and asks if the domain can be restricted to points with distance less than δ from x_0 so that the graph over that restricted domain lies between the two horizontal lines. If every challenge can be met, then the function is continuous at x_0 . In the case of a function of two variables, the challenge is the same, except that there are two horizontal planes at distance ε from the plane $z = f(x_0, y_0)$ and the responder has to choose δ so small that the graph over the disc of radius δ centered at (x_0, y_0) lies between the planes.

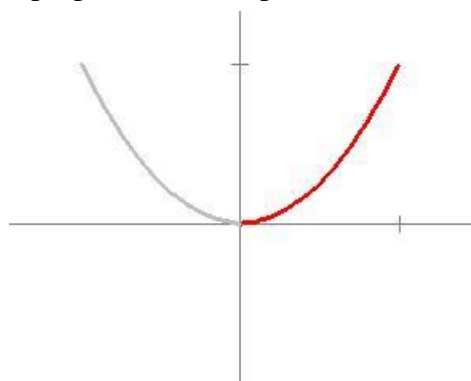


[D]

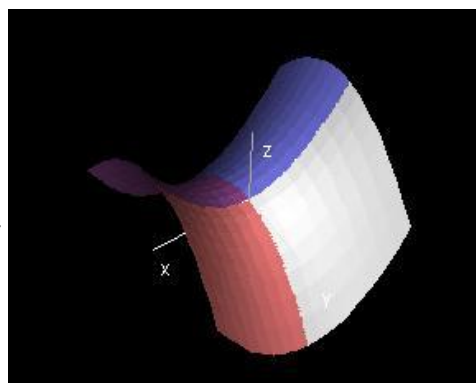


[D]

Another example refers to the author's paper in Volume 2, Number 2 of eJMT. Critical points of a function of one variable can be visualized by coloring the graph of a function $f(x)$ red if the derivative $f'(x)$ is positive. Similarly critical points of a function of two variables can be visualized by coloring the graph of a function $f(x,y)$ red if $f_x(x,y)$ is positive, blue if $f_y(x,y)$ is positive, and purple if both are positive.



[D]



[D]

An example appears in the interactive article [B1] concerning the topic of critical points. Students are familiar with Rolle's theorem for a function $f(x)$ of a single variable, stating that such a continuous function defined over an interval $a \leq b$ must have an interior local maximum or minimum or both if $f(a) = f(b)$. To generalize this, consider a continuous function $f(x,y)$ defined on a region D bounded by a curve C on which the function f is constant. The conclusion is that there must be a local maximum or a local minimum or both in the interior of D . If the function of one variable is differentiable, then the derivative of f must be zero at any interior local maximum or minimum so the tangent line is horizontal there. For a function of two variables, all partial derivatives at an interior local maximum or minimum must be zero and the tangent planes at such points must be horizontal. Students can then be challenged to state and prove generalizations of the Mean Value Theorem, where a differentiable function $f(x,y)$ coincides with a linear function $L(x,y) = ax + by + c$ on the boundary C curve of a disc domain D . More complex questions deal with the number of critical points of various types for functions

that are assumed to be non-negative on a domain and zero on the boundary. In the case of one variable, if the critical points are isolated maxima and minima, then the number of maxima is one greater than the number of minima. For the corresponding two-dimensional theorem, if all critical points are m_0 local minima, m_1 ordinary saddles, and m_2 local maxima, then students can explore and come up with the conjecture that $m_2 - m_1 + m_0 = 1$, a powerful and important result popularized by Marston Morse and reported in [M].

Interactive Laboratories: Under the Resources Menu in the course webpage, there is a link to the interactive laboratories developed using the senior thesis of David Eigen, a math-CS concentrator who devised the program for generating Java applets for use in a variety of mathematics courses at Brown. The Math 35 page includes a basic tutorial for using the Java demos. Each section of the labs starts with a labeled picture and a plus sign that can be selected to see more information about the concept illustrated in the demo as well as a button that opens a Java applet. The student can explore a particular phenomenon, entering in new data and manipulating the images. In order to share what he or she has discovered or created, the student can save the applet tag and enter it into a homework answer or a discussion. Anyone who opens the button that appears in the document will enter the program exactly where the student left off. The instructor or another student can then make comments, sometimes illustrated by a modification of the given applet.

3—Examples of Student Work Using the Tensor and Demonstration Software

Examples of such investigations from the Tensor appear in the work of Soravit Changpinyo and Thunwa Theerakarn, both students from Thailand in my honors multivariable class.

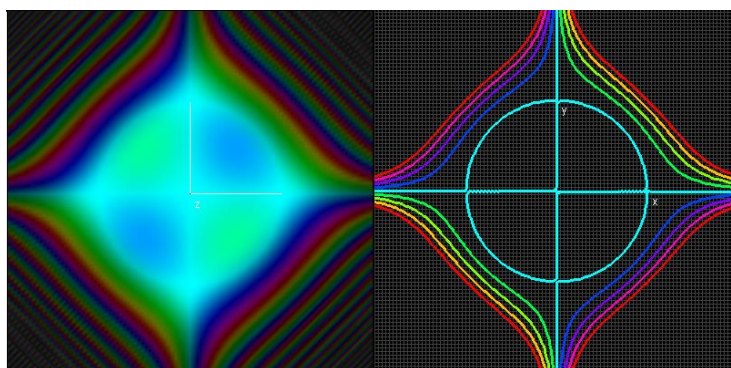
Problem: Analyze the function $f(x,y) = xy(1 - x^2 - y^2)$ showing the graph of the function together with its zero contour, and identify the critical points of the function. What is the maximum value taken on by the function on the domain consisting of all (x,y) with $x^2 + y^2 \leq 4$?

Soravit Changpinyo solves part (a) really well:

$$f(x,y) = xy(1-x^2-y^2)$$

$$f(x,y) = 0 \text{ when } x=0 \text{ or } y=0 \text{ or } x^2 + y^2 = 1$$

Its zero contour is along the light blue curve, two linear lines and a circle.



$$f_x(x,y) = xy(-2x) + (1-x^2-y^2)y = -3yx^2 + y - y^3$$

$$f_y(x,y) = xy(-2y) + (1-x^2-y^2)x = -3xy^2 + x - x^3$$

$$f_x(x,y) = -3yx^2 + y - y^3 = 0$$

$$y(1-3x^2-y^2) = 0$$

(1) $y = 0$ or

(2) $3x^2+y^2 = 1$

$$f_y(x,y) = -3xy^2 + x - x^3 = 0$$

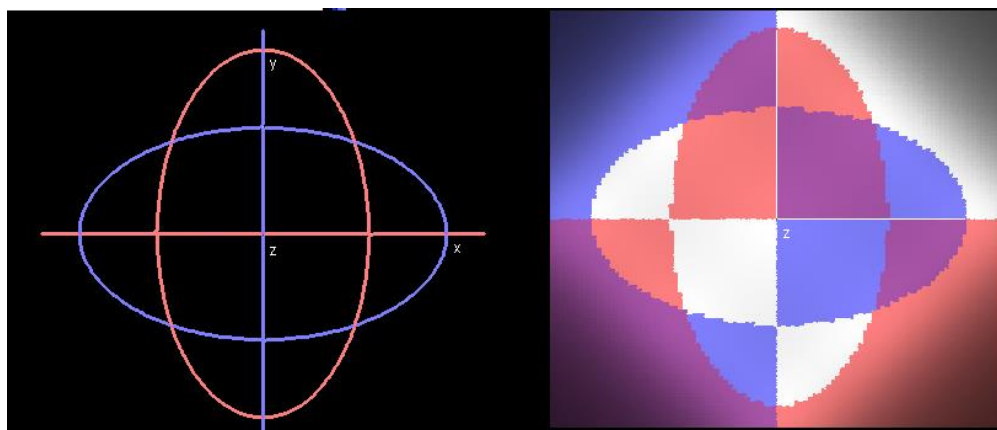
$$x(1-3y^2-x^2) = 0$$

(3) $x = 0$ or

(4) $3y^2+x^2 = 1$

The critical points are all (x,y) such that $f_x(x,y) = 0$ and $f_y(x,y) = 0$

They could be represented by the intersections of two ellipses ($3x^2 + y^2 = 1$ and $3y^2 + x^2 = 1$), the intersections of two lines (the x-axis and the y-axis), and the intersections of the line and the ellipse of different colors.



Solve for (x,y) from (1) and (3); $(0,0)$

Solve for (x,y) from (1) and (4); $(-1,0), (1,0)$

Solve for (x,y) from (2) and (3); $(0,1), (0,-1)$

Solve for (x,y) from (2) and (4); $(0.5,0.5), (-0.5,0.5), (0.5,-0.5), (-0.5,-0.5)$

The critical points are $(0,0,0), (-1,0,0), (1,0,0), (0,1,0), (0,-1,0), (0.5,0.5,0), (-0.5,0.5,0), (0.5,-0.5,0), (-0.5,-0.5,0)$.

$$0 \leq x^2 + y^2 \leq 4$$

$$-3 \leq 1 - x^2 - y^2 \leq 1$$

$$(|x|-|y|)^2 = x^2 - 2|xy| + y^2 \geq 0$$

$$2|xy| \geq 4$$

$$|xy| \leq 2$$
$$-2 \leq xy \leq 2$$

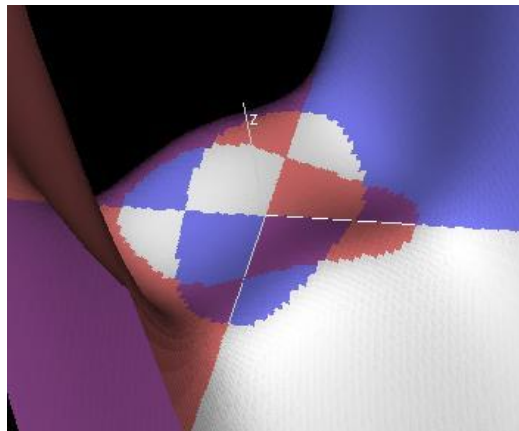
xy and $1 - x^2 - y^2$ must have the same sign so their product is positive.

if both xy and $1 - x^2 - y^2$ are positive, the possible maximum of the product is $(1)(2) = 2$.

if both xy and $1 - x^2 - y^2$ are negative, the possible maximum of the product is $(-2)(-3) = 6$.

The maximum value taken on by the function on the domain consisting of all (x,y) with $x^2 + y^2 \leq 4$ is 6.

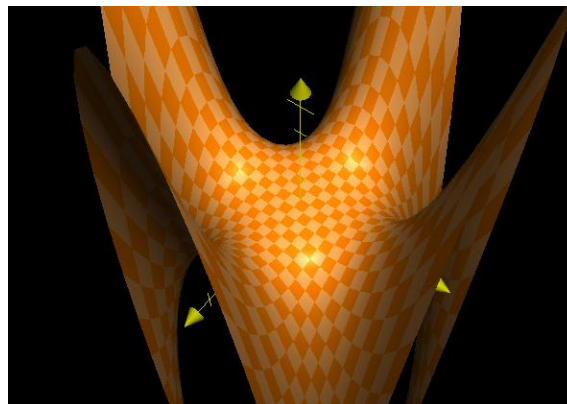
The demo below shows that there are two maximum points. Both of them satisfy $x^2 + y^2 = 4$.



[applet]

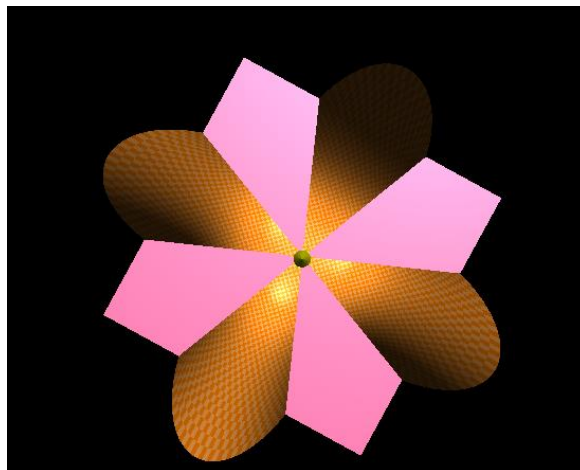
Problem: What can you say about the function $g(x,y) = x^4 - 6x^2y^2 + y^4$. (Standard Hint: look at the graph.)

Thunwa Theerakarn used standard Macintosh graphing hardware to produce images that he included in his assignments :



Graph of $g(x,y) = g(x,y) = x^4 - 6x^2y^2 + y^4$

With plane cut through $g(x,y) = 0$. The view is from above looking down to xy -plane.
 We will see that the plane divides the surface into 8 symmetrical pieces, 4 above and 4 below.



Make a conjecture that $g(x,y)$ can be written in $\cos(4\theta)$ form in polar coordinates.

Consider

$$\begin{aligned}
 g(x,y) &= x^4 - 6x^2y^2 + y^4 \\
 &= r^4\cos^4\theta - 6r^4\cos^2\theta\sin^2\theta + r^4\sin^4\theta \\
 &= [r^4\cos^4\theta - 2r^4\cos^2\theta\sin^2\theta + r^4\sin^4\theta] - 4r^4\cos^2\theta\sin^2\theta \\
 &= [r^4[\cos^2\theta - \sin^2\theta]^2] - r^4\sin^2(2\theta) \\
 &= r^4[\cos^2(2\theta)^2 - r^4\sin^2(2\theta)] \\
 &= r^4\cos(4\theta)
 \end{aligned}$$

Therefore, zero contour level is lines $\cos(4\theta) = 0$, that is, the lines $\theta = \pi/8, 3\pi/8, 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8, 13\pi/8, 15\pi/8$.

It might be interesting to some readers to look at the responses of these two students to the initial questionnaire for the course: In Soravit's Mathematical Autobiography, written the first day of the course, he states: "There are three things I expect from this class. The first thing is challenging stuff. Math that's too easy isn't fun. Another thing is a strong foundation in calculus that will ease my learning in computer science. Lastly, I would like to learn about math not only from the instructors, but also from my peers. I know everyone is exceptionally talented." He indicated in his questionnaire that my course had been recommended to him by Saran Ahuja, a member of the Thai Mathematical Olympiad team when he was a student at the Montford School in Chiangmai, and one of the top students in my course in Honors Linear Algebra during his first semester at Brown three years ago.

Thunwa writes "At the end of my high school years, I won a Thai Government scholarship to study in the United States from undergrad to Ph.D. The scholarship requires me to earn degree on Applied Mathematics and I will have to work in a university in Thailand for twice as time I will have spent here. This course will be hard - I know. Somebody told me I should not start first year in college with a hard course. But... there's so much to learn, and I'm willing to learn it. As always, it will be challenging, fun, and beautiful.

Problem: Find the points of the ellipse $x^2 + xy + y^2 = 1$ closest to the origin and furthest from the origin by finding the critical points of $f(x,y) = x^2 + y^2$ on the ellipse. (See the Hint to recall the procedure for Lagrange multipliers.)

Solution to Problem:

Let $g(x,y) = x^2 + xy + y^2$

$f(x,y) = x^2 + y^2$

Consider the minimum and maximum of $f(x,y)$ on level curve $g(x,y) = 1$

Using Lagrange multipliers,

$\nabla f(x,y) = \lambda \nabla g(x,y)$

$\nabla f(x,y) = (2x, 2y)$

$\nabla g(x,y) = (2x+y, 2y+x)$

$2x = \lambda(2x+y)$ and $2y = \lambda(2y+x)$

That is $2x/(2x + y) = 2y/(2y + x)$

that is $x^2 = y^2$

Case $x = y$

from $g(x,y)$ we get that $x^2 + x(x) + x^2 = 1$

That is $3x^2 = 1$

$x = 1/\sqrt{3}$

$(x,y) = (1/\sqrt{3}, 1/\sqrt{3}), (-1/\sqrt{3}, -1/\sqrt{3})$

$f(x,y) = x^2 + y^2 = 2/3$

Case $x = -y$

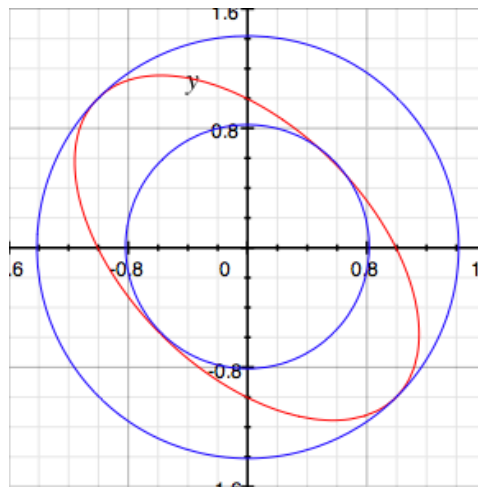
From $g(x,y) = 1$, we get that $x^2 = 1$

That is $x = \pm 1$

$(x,y) = (1,-1), (-1,1)$

$f(x,y) = x^2 + y^2 = 2$

Therefore, $g(x,y)$ closest to the origin at $(x,y) = (1/\sqrt{3}, 1/\sqrt{3}), (-1/\sqrt{3}, -1/\sqrt{3})$ and furthest at $(x,y) = (1,-1), (-1,1)$.



Problem: Find the critical points of the function $f(x,y,z) = x^2 + y^2 + z^2$ on the ellipsoid $x^2 + y^2/4 + z^2/9 = 1$ and indicate whether they are maxima or minima or other.

Here is a demo intended to give some inspiration. Changing r will alter the radius of an expanding sphere, representing the contour surfaces of $f(x,y,z)$. What happens when the sphere and the ellipsoid intersect?

[applet]

Solution to Problem:

$$f(x,y,z) = x^2 + y^2 + z^2$$

$$\text{Let } g(x,y,z) = x^2 + y^2/4 + z^2/9$$

Consider the critical points of $f(x,y,z)$ on the level set of $g(x,y,z) = 1$

Using Lagrange multipliers

$$\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$$

$$\text{That is } (2x+2y+2z) = \lambda(2x+y/2+2z/9)$$

implies

$$2x = \lambda 2x$$

$$2y = \lambda y/2$$

$$2z = \lambda 2z/9$$

That is

$$x = 0 \text{ or } \lambda = 1$$

$$y = 0 \text{ or } \lambda = 4$$

$$z = 0 \text{ or } \lambda = 9$$

Consider that in each case, λ cannot be the same constant. Therefore, two of $\{x,y,z\}$ have to equal zero in each case.

When $x=0, z=0, y=\pm 2$

$$f(x,y,z) = 4$$

When $x=0, y=0, z=\pm 3$

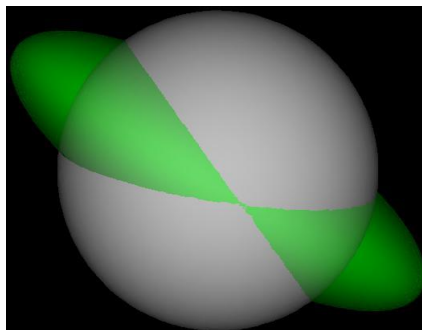
$$f(x,y,z) = 9$$

When $y=0, z=0, x=\pm 1$

$$f(x,y,z) = 1$$

Therefore, $(0,0,\pm 3)$ are maxima, $(\pm 1,0,0)$ are minima and $(0,\pm 2,0)$ are critical points but not maxima or minima.

At the critical points, the gradient vectors of f and g are parallel

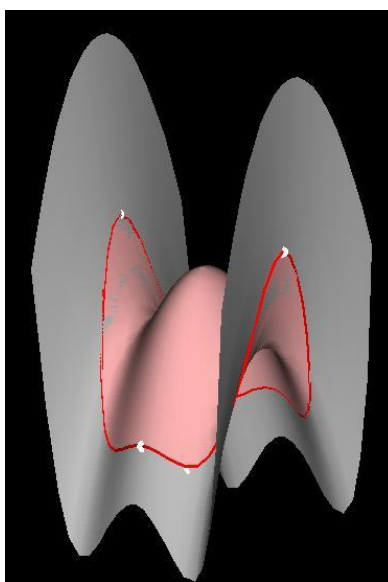


Critical Points at Level $f(x,y,z) = 2$

We take our final illustration from one of the problems on the midterm examination. The first examination in the course took place four weeks after the start of the semester. The Midterm Examination was a take-home test over a period of six days. It consisted of ten problems, each with two or three parts, and students were permitted to use notes and books and computers, as well as all the material on the class website, but they all agreed that no one would consult anyone else, either in person or electronically.

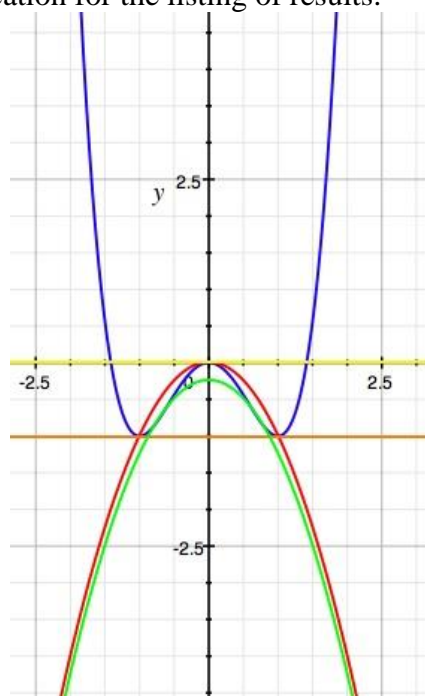
The first problem required students to identify the critical points of a polynomial function $f(x,y) = x^4 - 2x^2 - y^2$ and to find the maximum and minimum values on a disc of radius R centered at the origin, where the result depended on R .

The last part of the problem was subtle enough to test the ingenuity of most students. William Fallon devised a demonstration that was particularly useful in seeing how the image of a disk of radius R changed with R ,



[applet]

Gili Kriger was able to construct a two-dimensional graphic showing five different functions of R on the same diagram, from which it was possible to read off the required maxima and minima, and to provide algebraic justification for the listing of results.



4—Assessment and Future Directions Based on Comparison of Questionnaire Results

Recent proposals to the Division of Undergraduate Education of the National Science Foundation have placed special emphasis on developing effective assessments of the teaching and learning aspects of the project, particularly in the dissemination phase. Potential users of the technology need to know the results of our experience, and the ways in which we have modified the course, the technology, and the assessment instruments, so that they can determine whether and how this approach might be adapted to their needs and the needs of their students and institutions.

Course evaluation data have been collected in more than twenty courses using this approach, mostly at Brown University but also at Yale University, the University of Notre Dame, UCLA, and at the University of Georgia, both in the Mathematics department and in the College of Education. In 2004, we redesigned our questionnaires with the help of a professional market researcher who specializes in analysis of qualitative data. She has developed a method for systematically analyzing the data using a form of content analysis which enables us to summarize and display our results, course by course as well as for the same course across semesters and teaching situations (i.e., different teachers, different schools). These data collection techniques and methods of analyses represent a substantial improvement over our earlier assessment efforts.

For each course, the data were aggregated into question-by-question spreadsheets; responses for each question part, or sub-question, were coded (positive, neutral, and negative using a three-point scale) and displayed next to the individual verbatim response. Finally, summary tables

were generated for each question by sub-questions. In this method of analysis, some comments within verbatim responses could be highlighted for making summary points in the data tables, and for use in reports and presentations.

(Verbatim comments were only reported for students who had handed in a written statement giving the instructor permission to use submitted work in presentations and reports.) Examining coded verbatim responses for each question reveals patterns in students' responses within a course. Summary tables of codes by question and sub-question yields the more substantive analysis, and results for the same question or sub-parts of a question can be compared across time. Examining the data in these several systematic ways establishes a basis for deciding how to adapt and improve the teaching technology and interactive process for students and the teaching situation for the instructor.

Response rates for the two courses were comparable; in Fall 2004, 39 of 60 students completed course evaluations (**65%**), and in Fall 2008, course evaluations were received from 54 of 63 students who completed the course (**86%**).

In 2004, the questionnaire included nine questions (with up to four sub-parts), covering: 1) online assignments, 2) solution keys and hints 3) online student interaction 4) online communication with instructors 5) timing of assignments 6) examinations 7) textbook 8) visualization software ("demos"), and 9) an additional open-ended question allowing for general comments. In 2008, question 7 was altered since there was no specific text for the course, and a tenth question was added concerning the policy of working in groups for the bi-weekly homework assignments (a new procedure introduced in 2008).

In 2004, students submitted their course evaluations online after they completed classes and before completing their final exam, while in 2008, students submitted their course evaluations online just after completing a final take-home exam, with the understanding that the questionnaires would only be read after the grades had been determined.

Analysis of the Question on Comfort Level for Sharing Work Online

The value of this assessment method can be illustrated with an analysis of responses to an important question we asked about our teaching and learning approach, namely how "comfortable" students are with opening their work to other students online. This element of viewing the work of their classmates, and instructor comments is central to our model of teaching and learning. We also asked if students if their attitudes had changed over time. In the 2004 questionnaire, some students expressed discomfort at having their work on assignments and examinations, as well as instructor comments (but not grades) available for others to read. In subsequent courses, the software was modified so that students are given the option to designate any particular response "private", so only readable by instructors. Responses to the analysis of other questions have also led to modifications and refinements; we expect this to continue during the dissemination phase of our project.

Table 1. Learning from other Students

**Honors
Multivariable Calculus**

	Fall 2004	Fall 2008
a. How often did you look at the work of other students?		
<i>(Base: # Respondents)</i>	<i>(39) #</i>	<i>(54) #</i>
“Usually”	14	7
“Sometimes”/“occasionally”	15	19
“Rarely”/“hardly ever”	10	21
“Never”	0	6
<i>No answer</i>	0	1
b. Did you look at the work of some students more often than others? If so, how did you decide whose work to look at?		
<i>(Base: # Respondents)</i>	<i>(39) #</i>	<i>(54) #</i>
“No”	8	12
“Yes”	30	36
<i>No answer</i>	1	6
c. How comfortable were you with having your homework available for other students to read?		
<i>(Base: # Respondents)</i>	<i>(39) #</i>	<i>(54) #</i>
“Okay”/“no problem”	34	36
“Some reservation [<i>but</i>] no objection”	5	8
“Not comfortable”	0	3
<i>No answer</i>	0	7

d. How comfortable were you with having your exams available for other students to read?

	(Base: # Respondents)	(39) #	(54) #
“Okay”/“no problem”		30	28
“Some reservation [but] no objection”		9	12
“Not comfortable”		0	9
No answer		0	5

Several students reported changes in their attitudes over time. Most interesting is the student who said:

"In the beginning I really did not like the system of reading other student homework. It made me feel really nervous and exposed and it made me want to leave problems blank rather than to put in an incorrect answer. As the semester progressed I realized what a useful tool it could be and I started reading other people's hw responses more and more and felt more comfortable with mine being read."

Another student addressed motivation:

"It didn't really bother me that other people could look at my work, either at the beginning or the end of the semester. Sometimes I felt bad about the quality of the work that I handed in and I might have preferred that others not look at it, but it didn't concern me enough to make me want to change the system and the motivation to do a better job probably didn't hurt either."

As the numbers indicate, some students felt differently about exams:

"I think I am more self-conscious about my exams. At first I didn't mind so much, but now I am beginning to think that there should be some element of privacy with exams, or a way to choose to have your exam available or not".

A counter-intuitive response was:

"I have always been self-conscious about having work of mine open to criticism, so I was slightly uncomfortable (about the homework being available)" but "On exams, I was able to put more time into refining my answer, so I didn't mind having people see that."

Analysis of the Questions on Group Work for Assignments

A major change in the communication aspect of the course is the requirement that students participate in homework groups for their bi-weekly assignments, resulting in a manageable 20 or so responses instead of 66. Comfort levels on making homework responses public rose slightly, although most students reported sharing primarily within the group rather than reading other entries in the Tensor. A number of students appreciated selecting certain student answers for the Solution Key for each assignment. There was still some dissatisfaction with making examination answers available for everyone in the class to read. On the basis of this information, we will alter the software so that it will be possible for a student to designate the totality of his or her examination responses as private, rather than having to make this designation on each separate question.

Table 2. Sample Groups A and H

Briefly, describe the way that you worked [in your homework group]:							
		10.a	10.b	10.c			
Group	ID	a) Who was in your homework group? How often did you meet in person? How many attended in-person sessions? How did your meetings change over the semester?	b) What <i>problems</i> , if any, did you experience with the group process? Any suggestions for improving it?	c) How comfortable were you working in the group? Did your feelings change?			
A	amc	2	kmd, a person in my dorm. We met about twice a week, always in person.	OK	None really.	OK	It was very helpful for me at time. We sometimes thought up some clever stuff together.
A	kmd	2	I was in a group with amc. ... we would meet the night it was due and clear up any problems that we had. Usually ... about a half hour or so (occasionally much longer if we had lots of questions). As the course progressed I found that I typically was the one who was typing up the problems which was fine because that was how I like to do the problems anyway.	+/-	I felt as though groups can benefit certain people more than others. Typically one person will have more problems with the homework than others which means that one person will spend most of the time explaining to other people which is less beneficial for them.	OK	I felt very comfortable working in the group.
H	mlw	5	For the majority of the assignments I worked with [names] ... since I ... worked a lot with [names] in physics class and we worked well together. We usually met pretty consistently every Sunday and Wednesday in person to do the homework.	OK	The group worked great for me.	OK	In the beginning I was not excited about having to do the homework in groups, but it turned out to be very useful. Especially when the problems were conceptually difficult it was really beneficial having lots of people thinking about it and eventually talking it out rather than potentially getting stuck not starting a lot more problems. Having to explain

						our thinking seemed to make everyone grasp the material better since its a lot easier to write faulty logic than talk other people through it.
H	adp	5	Me, [names] ... We met once for each assignment we were allowed to do in groups, but not otherwise. Everyone showed up, and we'd split up the problems but everyone would check everyone else's work	OK	None except that it made the non-group process seem more difficult	OK consistently comfortable
H	bal	5	[names] ... were in my group. We met in person for almost every homework assignment towards the end of the semester. Not that frequently at the beginning. We all attended the sessions. Our meetings became more productive over the semester.	OK	The geographic organization of groups is a good starting point. It's much easier to work with people who you live near.	OK I was more comfortable towards the end of the semester. The first group meeting was awkward and we didn't work as well since we were just getting to know one another.
H	rpp	5	My core homework group consisted of [names] ... and myself. We met twice a week and usually worked out any problems we were having with the homework, then split the writing up of the problems between us. The homework group was a great asset to me; the other people in my group usually provided a fresh perspective on a problem that had become stagnant in my mind. The spreadsheet that listed students by their physical location was instrumental in arranging the groups; I would definitely make it a point to create one of these for all future classes.	OK	I had no problems whatsoever and very much enjoyed working with my group.	- [no answer]
H //Solo	jwb	5 //0	I rarely participated in my group as the rest of [residence hall] seemed to have a very different schedule from me.	-	[no answer]	+/- I liked it when I participated, but it wasn't really practical for me.

Table 3. Sample Summary Results on Group Work (Q10C)

How comfortable were you working in the group? Did your feelings change?

78% were comfortable working in the group

- 17 out of 54 -- 33% -- found their group's process "comfortable" and cited specific advantages.
- 15 out of 54 -- 28% -- said "comfortable" / "fine" / "no problem" / "liked it". One noted that group work "didn't help much with the math."
- 10 out of 54 respondents -- 19% -- got MORE comfortable with their group's process as the semester went on.

13% made negative comments about working in groups

- Most (4 out of 7) commented about "inefficiency".
- Only one person complained that group work meant "being asked to give others credit for my work".

5 out of 54 -- 9% -- did not participate in a group, or did not answer the question.

Responses to the question about group work suggest a number of modifications in the Tensor software that can facilitate further interaction. The author intends to work in the next stage of this project with colleagues Barbara Reynolds of Stritch University [R], an expert on group processes in mathematics instruction, and Thomas Cooper, who has designed and studied modifications of the Tensor software for enabling group interaction in pre-calculus courses in a non-residential college [C].

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